# Simple and Efficient Hard Label Black-box Adversarial Attacks in Low Query Budget Regimes

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Original Image Label: wolf



+



Adversarial Perturbation



Adversarial Image Label: shower curtain



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Attack Goal: Untargeted and Targeted



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- Distance Metrics:  $\ell_2, \ell_\infty$



- Attack Goal: Untargeted and Targeted
- · Distance Metrics:  $\ell_2, \ell_\infty$
- Threat Model: White-box and Black-box



<sup>&</sup>lt;sup>1</sup>picture taken from Chen et al. (2019)



• White-box Attacks

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- White-box Attacks
- Score-based Black-box Attacks (Soft Label)

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- White-box Attacks
- Score-based Black-box Attacks (Soft Label)
- · Decision-based Black-box Attacks (hard label)

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- Our proposed method uses Bayesian optimization for finding adversarial perturbations in low dimension subspace.
- Our proposed approach achieves higher attack success rate compared to the current state-of-the-art methods while requiring much fewer queries.

### **Problem Formulation**

#### $\cdot$ Notations

- Target Model  $F : \mathbb{R}^d \to \{1, \dots, K\}$
- original image  $x \in \mathbb{R}^d$
- Original label  $y \in \{1, \ldots, K\}$
- + Perturbation  $\delta$
- + Distance threshold  $\epsilon$

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- + Distance threshold  $\epsilon$
- $\cdot$  Objective

$$\max_{\delta} f(\mathbf{x}, y, \delta)$$
  
subject to  $\|\delta\|_{p} \le \epsilon$  and  $(\mathbf{x} + \delta) \in [0, 1]^{d}$   
where  $f(\mathbf{x}, y, \delta) = \begin{cases} 0 & \text{if } F(\mathbf{x} + \delta) \ne y \\ -1 & \text{if } F(\mathbf{x} + \delta) = y \end{cases}$ 

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- · Gaussian Processes as the surrogate model.
- Expected Improvement as the acquisition function.
- Running BO over high dimensional inputs such as ImageNet (3 × 299 × 299) practically infeasible

### Low Dimensional Subspace for $\ell_2$

- Bayes Attack utilizes low-frequency FFT basis vectors to generate  $\ell_2$  norm constrained adversarial perturbations.

FFT: 
$$X[u, v] = \frac{1}{d} \sum_{i=0}^{d-1} \sum_{j=0}^{d-1} x[i, j] \exp\left[-j\frac{2\pi}{d}(u \cdot i + v \cdot j)\right]$$
  
IFFT:  $x[i, j] = \frac{1}{d} \sum_{u=0}^{d-1} \sum_{v=0}^{d-1} X[u, v] \exp\left[j\frac{2\pi}{d}(u \cdot i + v \cdot j)\right]$ 

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- Equivalent norm:  $\|\mathbf{x}\|_2 = \|\mathsf{FFT}(\mathbf{x})\|_2$ ,  $\|\mathbf{X}\|_2 = \|\mathsf{IFFT}(\mathbf{X})\|_2$
- To allow only low-frequencies, the top-left  $\lfloor rd \rfloor \times \lfloor rd \rfloor$ ,  $r \in (0, 1]$  square of X have nonzero entries



- Bayes Attack utilizes spatial local similarity in images to generate  $\ell_{\infty}$  norm constrained adversarial perturbations.

## Low Resolution Subspace for $\ell_\infty$

- Bayes Attack utilizes spatial local similarity in images to generate  $\ell_\infty$  norm constrained adversarial perturbations.
- We search for perturbations in a lower resolution image space  $\lfloor rd \rfloor \times \lfloor rd \rfloor, r \in (0, 1]$  and use nearest neighbor interpolation.
- + Equivalent norms:  $\left\|\mathbf{X}\right\|_{\infty} = \left\|\mathsf{NNI}(\mathbf{X})\right\|_{\infty}$



- 1: **procedure** BAYES-ATTACK $(x_0, y_0)$
- 2:  $\mathcal{D} = \{(\boldsymbol{\delta}_1, \boldsymbol{v}_1), \cdots, (\boldsymbol{\delta}_{n_0}, \boldsymbol{v}_{n_0})\}$

 $\triangleright$  Quering randomly chosen  $n_0$  points.

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- 2:  $\mathcal{D} = \{(\boldsymbol{\delta}_1, \boldsymbol{v}_1), \cdots, (\boldsymbol{\delta}_{n_0}, \boldsymbol{v}_{n_0})\}$
- 3: Update the GP on  $\mathcal{D}$
- 4:  $t \leftarrow n_0$
- 5: while  $t \le T$  do
- 6:  $\delta_t \leftarrow \operatorname{arg\,max}_{\delta} \mathcal{A}(\delta \mid \mathcal{D})$

Quering randomly chosen n<sub>0</sub> points.
 Updating posterior distribution
 Updating number of queries

▷ Optimizing the acquisition function

10:  $t \leftarrow t + 1$ 

- 11: **if**  $v_t < 0$  **then**
- 12:  $\mathcal{D} \leftarrow \mathcal{D} \cup (\delta_t, v_t)$  and update the GP  $\triangleright$  Updating posterior distribution

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  - $\boldsymbol{\delta}_t \leftarrow \boldsymbol{\Pi}^p_{B(\mathbf{0},\epsilon)}(\boldsymbol{\delta}_t)$
- 8:  $\Delta_t \leftarrow map(\delta_t)$

9: 
$$v_t \leftarrow f(\mathbf{x}_0, y_0, \boldsymbol{\Delta}_t)$$

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▷ Quering randomly chosen n₀ points.
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▷ Optimizing the acquisition function
 ▷ Projecting perturbation on ℓ<sub>p</sub>-ball
 ▷ Mapping perturbation to full input space
 ▷ Querying the model

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3:	Update the GP on ${\cal D}$	Updating posterior distribution
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5:	while $t \leq T$ do	
6:	$oldsymbol{\delta}_t \gets argmax_{oldsymbol{\delta}}  \mathcal{A}(oldsymbol{\delta} \mid \mathcal{D})$	<ul> <li>Optimizing the acquisition function</li> </ul>
7:	$\boldsymbol{\delta}_t \leftarrow \Pi^p_{B(0,\epsilon)}(\boldsymbol{\delta}_t)$	$\triangleright$ Projecting perturbation on $\ell_p$ -ball
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10:	$t \leftarrow t + 1$	
11:	if $v_t < 0$ then	
12:	$\mathcal{D} \leftarrow \mathcal{D} \cup (oldsymbol{\delta}_t, v_t)$ and upd	ate the GP ▷ Updating posterior distribution
13:	else	
14:	return $oldsymbol{\delta}_t$	Adversarial attack successful
15:	return $\delta_t$	Adversarial attack unsuccessful

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                                                                        \triangleright Quering randomly chosen n_0 points.
 3:
       Update the GP on {\cal D}
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 4.
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Untargeted and targeted attacks

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- MNIST, CIFAR-10 4 convolution and 2 fully-connected layers
- ImageNet ResNet50, VGG16-bn, Inception-v3

- Untargeted and targeted attacks
- +  $\ell_2$  and  $\ell_\infty$  threat models
- MNIST, CIFAR-10 4 convolution and 2 fully-connected layers
- ImageNet ResNet50, VGG16-bn, Inception-v3
- Compare with Boundary attack<sup>2</sup>, OPT<sup>3</sup>, Sign-OPT<sup>4</sup> and HSJA<sup>5</sup>

<sup>&</sup>lt;sup>2</sup>Wieland Brendel, Jonas Rauber, and Matthias Bethge. 2017. Decision-based adversarial attacks: Reliable attacks against black-box machine learning models. arXiv preprint arXiv:1712.04248 (2017).

<sup>&</sup>lt;sup>3</sup> Minhao Cheng, Thong Le, Pin-Yu Chen, Jinfeng Yi, Huan Zhang, and Cho-Jui Hsieh. 2019. Query-efficient hard-label black-box attack: An optimization-based approach. In International Conference on Learning Representations.

<sup>&</sup>lt;sup>4</sup> Minhao Cheng, Simranjit Singh, Patrick H. Chen, Pin-Yu Chen, Sijia Liu, and Cho-Jui Hsieh. 2020. Sign-OPT: A Query-Efficient Hard-label Adversarial Attack. In International Conference on Learning Representations.

<sup>&</sup>lt;sup>5</sup> Jianbo Chen, Michael I. Jordan, and Martin J. Wainwright. 2019. HopSkipJumpAttack: A Query-Efficient Decision-Based Attack. ArXiv abs/1904.02144 (2019).

 $\epsilon =$  0.05, Query budget = 1000

	ResNet50		Inception-v3		VGG16-bn	
Method	success	avg. query	success	avg. query	success	avg. query
OPT attack	5.73	246.31	2.87	332.17	7.53	251.21
Sign-OPT	10.31	660.37	7.51	706.3	15.85	666.87
Bayes attack	67.48	45.94	44.29	72.31	78.47	33.7

#### MNIST ( $\epsilon = 0.3$ ) and CIFAR-10 ( $\epsilon = 0.05$ ), Query budget = 1000.

Method	MNIST		CIFAR-10		
	success	avg. query	success	avg. query	
OPT attack	2.91	657.93	12.55	271.24	
Sign-OPT	7.02	682.36	31.87	679.39	
Bayes attack	90.35	27.56	70.38	75.88	

MNIST ( $\epsilon = 0.3$ ) and CIFAR-10 ( $\epsilon = 0.1$ ), Query budget = 1000.

Method	MNIST		CIFAR-10		
	success	avg. query	success	avg. query	
OPT attack	0.0	_	0.0	_	
Sign-OPT	2.41	975.67	3.50	937.65	
Bayes attack	26.23	130.03	48.93	149.15	

## Untargeted $\ell_2$ attacks on ImageNet



### Query Efficiency Comparison



(a) MNIST

#### **Query Efficiency Comparison**



Performance comparison of FFT basis vectors and random vectors sampled from the standard normal distribution for  $\ell_2$  attack with  $\epsilon = 20.0$  on ResNet50.

Basis	Success	Avg Queries
Cosine FFT	64.38%	54.25
Sine FFT	63.74%	45.72
Cosine and sine FFT	66.67%	54.97
Standard Normal	33.33%	48.25

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- We show that BO presents as a scalable, query-efficient alternative for black-box adversarial attacks when combined with searching in structured low dimensional subspaces.
- We successfully demonstrate the efficacy of our method in attacking multiple deep learning architectures in both untargeted and targeted settings, and  $\ell_{\infty}$  and  $\ell_2$  norms.

# Thank You.

- Poster Session: 18<sup>th</sup> August 2021, 05:30 pm 08:30 pm EST
- · Code: https://github.com/satyanshukla/bayes\_attack
- Paper: https://arxiv.org/pdf/2007.07210.pdf
- · Contact: snshukla@cs.umass.edu